

Heights of rivers above zeros of gauges—Continued.

Stations.	Distance to mouth of river.	Danger-line on gauge.	Highest water.		Lowest water.		Mean stage.	Monthly range.
			Height.	Date.	Height.	Date.		
<i>James River.</i>	<i>Miles.</i>	<i>Feet.</i>	<i>Feet.</i>		<i>Feet.</i>		<i>Feet.</i>	<i>Feet.</i>
Lynchburg, Va.	257	18	1.9	23	0.1	17-19	0.6	1.8
Richmond, Va.	110	12	0.7	24	-0.2	2, 9, 10, 12	0.2	0.9
<i>Alabama River.</i>								
Montgomery, Ala.	265	25	8.4	24	0.5	2	2.8	7.9
Selma, Ala.	212	35	9.8	25	0.5	3	3.3	9.3
<i>Coosa River.</i>								
Gadsden, Ala.	144	18	10.1	22	0.0	1	2.6	10.1
<i>Tombigbee River.</i>								
Columbus, Miss.	285	33	-0.2	17	-3.0	31	-2.3	2.8
Demopolis, Ala.	155	35	8.2	23	-1.5	2-6	1.2	3.7
<i>Black Warrior River.</i>								
Tuscaloosa, Ala.	90	38	14.5	21	-0.2	2-4	3.4	14.7
<i>Pedee River.</i>								
Cheraw, S. C.	145	27	20.4	23	2.0	31	5.4	18.4
<i>Black River.</i>								
Kingstree, S. C.	60	12	3.7	30, 31	1.3	11-13	2.5	2.4
<i>Lumber River.</i>								
Fair Bluff, N. C.	10	6	5.1	31	-0.4	9	1.7	5.5
<i>Lynch Creek.</i>								
Effingham, S. C.	35	12	12.1	29	2.5	7, 8	5.2	9.6
<i>Potomac River.</i>								
Harpers Ferry, W. Va.	170	16	2.0	28	0.1	19	1.1	1.9
<i>Roanoke River.</i>								
Clarksville, Va.	155	12						

Heights of rivers above zeros of gauges—Continued.

Stations.	Distance to mouth of river.	Danger-line on gauge.	Highest water.		Lowest water.		Mean stage.	Monthly range.
			Height.	Date.	Height.	Date.		
<i>Sacramento River.</i>	<i>Miles.</i>	<i>Feet.</i>	<i>Feet.</i>		<i>Feet.</i>		<i>Feet.</i>	<i>Feet.</i>
Red Bluff, Cal.	241	23	1.0	1-5	0.1	23-31	0.5	0.9
Sacramento, Cal.	70	25	13.8	1	10.0	31	11.6	3.8
<i>Santee River.</i>								
St. Stephens, S. C.	50	12	7.3	28-31	2.6	7	5.3	4.7
<i>Congaree River.</i>								
Columbia, S. C.	37	15	3.5	21, 22	1.5	{ 1-13, 15-19, 24-31 }	1.7	2.0
<i>Wateree River.</i>								
Camden, S. C.	45	24	22.5	22	4.0	2-4, 8	7.4	18.5
<i>Savannah River.</i>								
Augusta, Ga.	180	32	16.6	20	5.7	17	8.5	10.9
<i>Susquehanna River.</i>								
Wilkesbarre, Pa.	178	14	5.0	31	-1.0	1-27	-0.5	6.0
Harrisburg, Pa.	70	17	4.5	30	1.0	13, 15, 16	1.6	3.5
<i>Juniata River.</i>								
Huntingdon, Pa.	80	24	4.0	19, 23, 29	2.8	10-18	3.1	1.2
<i>W. Br. of Susquehanna.</i>								
Williamsport, Pa.	35	20	4.8	30	0.7	6-9	1.5	4.1
<i>Waccamaw River.</i>								
Conway, S. C.	40	7	2.2	7, 30	0.6	26	1.6	1.6

*Distance to the Gulf of Mexico. †Record for 28 days

SPECIAL CONTRIBUTIONS.

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THE OBSERVATION OF HALO PHENOMENA.¹

(Translated from a separate print from the annual volume of the Natural History Association of Wurtemberg. Communicated by Rev. K. SCHEFFÉ; dated Feb., 1897.)

The light from the sun, moon, and brighter stars, by means of refraction through or reflection on ice crystals—when the latter occur in great numbers, in definite positions, over a considerable region—manifests itself in figures of manifold forms, known as halo phenomena. These are very seldom observed in crystals of ice that lie upon the surface of the earth; more frequently, but still rarely, in those crystals that float in the atmosphere in the immediate neighborhood of the observer so that, for instance, they rarely develop between the observer and any distant object. As a rule, and, indeed, by no means so rarely as is ordinarily thought, halos occur in the clouds or haze of ice crystals at different altitudes in the atmosphere, but above the observer's head. The most frequent form of halo is a circle around the star whose light produces it, having a radius of about 23°; that is to say, the line from the eye to the star makes an angle of about 22° to 23° with the line from the eye to the circle. The circle shows the colors of the rainbow, beginning with red on the inside of the circle (in contrast to the rainbow, where the red is on the

¹The Chief of the Weather Bureau has just received from Rev. K. Schipps, of Baustetten, near Laupheim, Wurtemberg, Germany, a letter requesting the cooperation of those interested in the study of halos. A committee for this purpose has been formed in Germany, on behalf of which Mr. Schipps has issued a circular, which we translate herewith, and which will be found instructive as a guide to both observers and students.

convex side). Most frequently we observe only the red on the concave side, but next frequently, also, a blue tint on the convex side. The circle has about the same breadth as that of the moon. The inner edge, viz, that turned toward the star, is without exception more sharply defined than the outer edge; the region between the halo and the star is peculiarly free from light, and almost always appears considerably darker than the area outside of the circle. The halo, like all forms of this phenomenon, is generally developed only in fragments, with a special preference for the upper region; that is to say, on the side nearer the zenith. Next in frequency after the halo are the mock suns, parhelia, and mock moons, which occur either in pairs or individually at the same altitude as the sun or moon and at the same distance, viz, about 23° , from it; therefore, approximately, lying on the circle just mentioned, generally not sharply defined and ordinarily of a deep purple red, especially on the side toward the luminary.

Concentric with the first-mentioned circle, there sometimes forms a larger one, with a radius of about 46° . Through the sun, parallel to the horizon, and therefore having its center in the zenith, there passes the white "parhelic circle," with two, four, or six mock suns. Ordinarily this parhelic circle is well developed only in the neighborhood of the first halo circle and the mock suns. Through the sun, perpendicular to the horizon, there is, also, a whitish band, called the "vertical column of light." Not infrequently there occurs in the upper part of the circle around the star a "tangential arc," located symmetrically to the vertical column of light, like a fragment of a circle of larger or smaller radius than the existing halo. Besides these many other forms occasionally occur (several dozen have been recorded), the individual description of which would lead us too far.

Halo phenomena have been very rarely observed about the brighter stars. Those around the moon are most frequent and easiest to see. Those of the greatest absolute frequency are the solar halos, but these are generally difficult to observe on account of the more intense sunlight and are overlooked, especially because they generally occur only in fragments.

Solar halos can, with advantage, be observed according to a method practiced in the earliest times by their reflection in water, or by reflection from smoked glass. But the observer should practice the direct observation of the sky. To this end, one may place himself in the shadow of a house or the thick foliage of a tree so that the direct sunlight is cut off. One will soon acquire so much experience that the hand or any other object held in front will suffice to cut off the sun's rays. It is best to first examine a region of the sky lying farther from the sun by the use of the above-mentioned primitive screens, and then by slow motion approach the region that is to be observed. A good eye possesses such a power of accommodation that it gradually becomes able to do without this assistance, at least for short intervals of time; still, care should be taken to preserve the eye from harm.

If a halo phenomenon is observed it should be immediately and accurately described so that the individual phases of the phenomenon should not be confounded. The description should include the name of the observer and of the locality, the time and the duration, expressed in central European standard time (one hour east of Greenwich¹). The form of the phenomenon is quickest and safest described by means of a simple drawing, which is best made by considering the top of the drawing as corresponding to the zenith. In the description, which should be as brief as possible, the ordinary phenomena can be described by simply using the name of the form. The orientation or location of the details with respect

to the sun or moon can ordinarily be given by using the letter *a* for above, *b* for below, *r* right hand, as seen by the observer, *l* left hand; *ra* for the region on the right hand and above the altitude of the sun; *rb* for the right hand and below; *la* left hand and above; *lb* left hand and below. The intensity, the location and order of sequence of the colors, as well as the general intensity of the light, in the several parts of the halo, should also be given. For this purpose the ordinary three scale, 0, 1, 2, is sufficient, but the 1 may be omitted, so that feeble, or intense, optical phenomena can be briefly sketched by writing the superscript 0, or 2, to the right hand of the letter *I* for intensity, so that *I*⁰ designates feeble intensity and *I*² designates very intense. In a similar way the nature of the boundary of the circle can be noted by using *B* for boundary and indicating by *Bi* the inner or concave and *Be* the exterior or convex side, to which the superscript 0 or 2 may be added for indefinite or very sharp edges, respectively; thus, *Bi*²=boundary of the side of the circle on the side toward the sun sharply defined, or *Be*⁰=boundary of the outer rim of the circle ill defined.

The symbol used in meteorology for all forms of these phenomena is, for solar halos, the well-known solar wheel \oplus of the ancient mythology. For lunar halos the upper half only of this same wheel \odot is used.

After some experience one will be able to decide quickly on the first glance at the sky whether or not a halo is to be expected within a given time, whereby on the one hand much time is saved, and on the other hand more phenomena are observed. When the cloudiness is favorable for the formation of halos one can not examine the sky too often on account of the rapid changes of the individual forms with location and with time. When the sky is covered only with cumulus clouds, in a state of rapid change of form and very sensitive to the influence of the solar radiation and through which the sun appears as a disk, it is certain that no halo will be formed. On the other hand if the sky is covered with small cirrus clouds, or little wool flecks, which in certain places thin out into grayish white streaks of perfectly homogeneous structure, or show themselves fantastically intertwined and drawn out as homogeneous shreds of clouds of small dimensions, as cats' tails, or such bands as radiating from one point pass over the whole sky, namely, polar bands, then certainly either a halo, or at least a fragment of a halo, or individual "mock suns" are to be expected. If these stripes broaden out and gradually cover the whole sky while at the same time the barometer falls, then more enduring, more complete, and often very complex forms of the phenomena are to be expected, until gradually the veil of clouds attains a dark gray black appearance and grows thicker, so that the sun is seen through it only as a bright spot without a definite border. From this time on a disappearance of the phenomena is again to be expected.

Whoever is in possession of the proper instruments should not neglect to take measurements of the angular distances of the phenomena from the sun. In doing this it is advisable not to stop with one measurement of the individual phenomena, but to take many of them in rapid succession, and furthermore to again undertake new measurements after the lapse of a considerable interval. From a morphological point of view (viz, the description of the forms) the halo phenomena have been rather thoroughly studied and observed. On the other hand, there is still wanting a long continued series of measurements, especially for our latitudes, in order to understand the changes that are going on.

Moreover, notwithstanding the extended network of meteorological stations, there are still wanting simultaneous observations over a large region since, as it would appear, meteorological optics as yet receives no great attention on the part of observers, except in Japan. But it is precisely these ob-

¹The observers of the United States Weather Bureau will, of course, use eastern or seventy-fifth meridian standard time.

servations that would be of value for the furtherance of our knowledge as to the clouds of ice crystals in front of the whirlwind or as to the local similar and simultaneous formation of ice in the atmosphere.

Even the individuality of the observer has an influence upon the minute details of observations that is of an importance not to be underestimated, and, therefore, every observer should seek to understand the errors to which he is personally liable, and the times at which he is in the best personal condition. This has a special bearing on the perception and distinction between colors. To one person great differences of intensity will alone affect his consciousness, whereas another has the power of distinguishing the finest shades; therefore, a note upon the individual colors that are observed and their succession from the inside to the outside of the halo should not be neglected. Not less important is the difference in the observation of the blue tints on the outer side of the circle, since one observer can observe violet only in a narrow zone, whereas another can follow it throughout a wider band.

In conclusion the subscriber invites all who have sufficient leisure and love of nature to join in the minutest observations of halo phenomena, and to kindly send him the results, since the greatest usefulness is to be expected from the discussion of all the material from a single point of view, and since, moreover, individual observations are easily scattered and lost.

THE EQUATIONS OF HYDRODYNAMICS IN A FORM SUITABLE FOR APPLICATION TO PROBLEMS CONNECTED WITH THE MOVEMENTS OF THE EARTH'S ATMOSPHERE.

By JOSEPH COTTIER, Columbia University, N. Y. (dated June 29, 1897).

Introductory note.—The equations of fluid motion as usually given in terms of rectangular coordinates are unsuitable for use in problems connected with the motions of fluids on the earth's surface, owing to the curved surface of the latter. Hence, either the Cartesian equations must be transformed into the equations for polar coordinates, by purely mathematical considerations, or an independent deduction of the equations in terms of polar coordinates must be made. The former course was adopted by Mr. Basset¹ in obtaining the equations of an incompressible fluid in terms of polar coordinates; the latter process is that adopted in this paper, longer and more laborious perhaps, but having the advantage of giving a clearer conception of the terms entering the final equations and of the intermediate steps.

The plan of this article follows closely that of a paper read by the writer before the American Mathematical Society,² adapted to the simpler polar coordinates. Since it is hoped this latter paper will soon be published, many of the more complex reductions therein have been dismissed with a mere reference, as being unnecessary to a comprehension of the equations and methods here employed.

General considerations.—The earth is a slightly ellipsoidal body, whose mean radius is about 3,959 miles, or 6,371 kilometers; the polar diameter is, roughly, only one three-hundredth part shorter than the mean equatorial diameter, so that all necessary accuracy will be obtained by considering the surface of the earth as a sphere of the above radius.

The coordinate system suitable for rotating spherical surfaces, or for surfaces departing but little from the spherical form, is, of course, the well-known "polar coordinate" system, where the coordinates determining the position of a

point are its north polar distance (θ), its rotation angle measured eastward (λ), and its distance from the center of the earth (r). The measurement of geographic longitude positive westward is determined by the fact of the eastward rotation of the earth in space.

The equations of motion which it is desired to obtain are three equations, which for three separate orthogonal directions express the equality of the "expressed force" of a fluid particle to the sum of the "impressed forces" and of the resultant surface traction on the particle in the same direction.

The "expressed force" of a particle is the force necessary to produce the existent acceleration in the given direction, and is equal to the product of the mass of the particle and of that acceleration.

The "impressed forces" are those forces which are applied individually to each particle of the fluid, such as gravitational forces, electrical forces, etc. The only impressed force which need be taken into account in connection with atmospheric disturbances is the gravitational attraction of the earth, whose value per unit mass in any direction may be expressed as the derivative in that direction of the Newtonian potential, F . It may be considered that the derivatives of F with respect to θ and λ , that is to say, parallel to the surface of the earth, are each zero.

The surface tractions are due in part to the "fluid pressure," constant in all directions at a given point and independent of the viscosity of the fluid, and in part to the internal friction of the fluid determined by the relative motions of its particles.

In all material fluids the internal friction or viscosity can not change the mean normal pressure, or the fluid pressure at any point, but it does cause a tangential resistance, or "shearing stress," between layers of particles which are in relative motion. It is generally assumed, and the assumption is borne out by experience, that the shearing stress at any point and parallel to any plane, is proportional to the rate at which the velocity parallel to the plane is varying in a direction normal to that plane, that is to say, viscosity is proportional to that component of the relative motion of the particles that is parallel to that plane, or to the rate of "shearing stress" upon it. A finite slip between the fluid and the surfaces with which it may be in contact, or between contiguous layers of the fluid itself, or between the layers at the limiting surface that is technically known as a "surface of discontinuity," would thus mean an infinite shearing stress parallel to the surface of discontinuity, and is hence precluded.

The coefficient of friction expresses the ratio between the shearing stress and the shearing strain; it may be defined conveniently as the difference of the tangential tractions per unit area on two very large and parallel planes at the unit distance apart, moving in the same direction with the unit difference of velocity. The space between these parallel planes being filled with the fluid, the velocity gradient in a direction normal to the slower moving plane will be unity, and therefore, according to definition, the intensity of the tangential stress on that plane due to viscosity, will be numerically equal to the coefficient of viscosity, which by common consent is denoted by the Greek letter μ . The coefficient of viscosity is, approximately, proportional directly to the absolute temperature of the air, and is independent of the pressure.¹ As nearly as present experimental data permit of its estimation, the value of the coefficient of viscosity at a temperature of 62° F., in terms of British units,² may be considered as being:

¹Maxwell, Phil. Trans., 1863.

¹Treatise on Hydrodynamics. Vol. II. §470.

²"On the Expression of the General Equations of Hydrodynamics in Terms of Curvilinear Coordinates," read before the American Mathematical Society, March 27, 1897, and about to be published in The Mathematical Review.

²The British units referred to are the foot and mean solar second as units of length and time, and the mass of a pound of matter as the unit of mass. The unit of force, or "poundal," is the force which, acting on a unit mass, produces unit acceleration.